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PRESSURE EXCURSIONS IN TRANSIENT FILM BOILING FROM A SPHERE

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Abstract—The pressure excursions accompanying the transient film boiling which occurs when a large pool of saturated and stagnant liquid is suddenly exposed to a very hot solid sphere are analytically evaluated. Following simplifying assumptions, it is shown that quite large pressure excursions can result. Predictive equations for their amplitudes and frequencies of oscillation are presented.

NOMENCLATURE

A, parameter, $A^2 = P_\infty/R\rho\delta_0$;
a, constant, $a = 2^{2/3}\Gamma(2/3)/3$;
B, parameter, $B = \delta_0/R$;
b, coefficient;
C, parameter, $C = k(T_w - T_s)\rho_v/P_\infty\Lambda$;
c, constant of integration;
F, function;
f, function of time;
J, Bessel function;
K, parameter, $K = k(T_w - T_s)/\rho_v\delta_0^2\Lambda$;
k, vapor thermal conductivity;
M, vapor mass per unit area in film;
m, dimensionless vapor mass per unit area, $m = M/M_0$;
P, local pressure;
P_∞, pressure far from sphere;
p, dimensionless film pressure, $p = P/P_\infty$;
R, sphere radius;
r, radius;
T_s, liquid saturation temperature;
T_w, sphere temperature;
t, time;
v, liquid velocity;
y, dimensionless variable, $y = \log_e(p)$;
z, dimensionless variable, $z = 2m^{3/2}/3\alpha$.

Greek symbols

α, dimensionless parameter, $\alpha = [k(T_w - T_s)/\rho_v\Lambda](R\rho/\delta_0^3 P_\infty)^{1/2}$;
Γ, gamma function;
δ, film thickness;
Λ, heat of vaporization;
ρ, liquid density;
ρ_v, vapor density at ambient pressure;
τ, dimensionless time, $\tau = At$.

Superscripts

$\dot{}$, first ordinary time derivative;
$\ddot{}$, second ordinary time derivative.

Subscripts

m, exponent;
n, index;
0, initial value.

INTRODUCTION

WHEN a very hot solid of constant temperature is suddenly brought into contact with a large pool of stagnant and saturated liquid, heat flows from the solid into the liquid and causes the liquid to vaporize. Thus, an ever increasing amount of vapor forms a film between the solid and the liquid. Because the vapor is much less dense than is the liquid, the liquid must ultimately be displaced away from the solid to make room for the vapor. Before such a displacement can occur, there must be a pressure excursion in the vapor film to provide the accelerating force. Inasmuch as the proper operation of thermal equipment requires that transient effects in change-of-phase processes be understood for appropriate precautions and controls to be employed, detailed understanding of this physical situation is believed to be pertinent to today's technology.

It appears, however, that no treatment directly applicable to the subject problem has been published. Most studies neglect the possibility of a pressure excursion in the film and its effect upon vapor density. Carslaw and Jaeger [1], Hamill and Bankoff [2], Pitts *et al.* [3], and Limpiyakorn and Burmeister [4] considered the effects of different geometries and temperature dependent properties and found the square of the film thickness to vary linearly with time. But this result, generally expressible as $\delta^2 = \delta_0^2 + ct$, gives $\dot{\delta}(t = 0) = c/2\delta_0$ and shows that under initially stagnant conditions there must have been large accelerations and pressure excursions in the beginning. Studies accounting for pressure effects on unsteady change-of-phase processes are few. The influence of externally imposed pressure variations on quasi-steady film boiling was analytically studied by Burmeister and Schoenhals [5] for a vertical plate and was experimentally studied by McCoy [6] for nucleate boiling on a horizontal wire. They both found an appreciable influence, but did not treat the

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case where pressurization is caused by the transient vaporization itself.

In the present work the amplitudes and frequencies of the pressure excursions and their effect upon the rate of vaporization and the vapor film thickness will be determined for representative conditions. It will be shown that pressure excursions can be extreme.

PROBLEM FORMULATION

As shown in Fig. 1, a sphere is immersed in a large pool of stagnant and saturated liquid with a very thin film of vapor initially separating the solid from the liquid. Suddenly, the sphere achieves a very high temperature which is thereafter maintained. Heat flows from the sphere into the liquid–vapor interface by conduction through the vapor, generating additional vapor. The pressure in the compressible film rises since the liquid is initially stationary, and the liquid accelerates away from the sphere. Because of its inertia the liquid undergoes too large a displacement; the pressure in the film decreases below the ambient value, and the liquid then accelerates toward the sphere. As a result, film thickness and pressure have an oscillatory behavior.

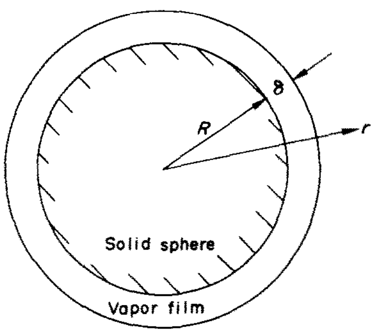

FIG. 1. Physical configuration and coordinate system.

A spherical geometry and a nonzero initial film thickness are considered to avoid the infinitely large pressure excursions which result from other geometries and the absence of an initial film. Gravitational body forces are neglected. It is assumed that the liquid and vapor are in equilibrium at the constant saturation temperature which corresponds to the initial pressure, neglecting the conductive heat flow into the liquid caused by the pressure dependence of saturation temperature which is expected to be of greater importance for large pressure excursions than for small ones. The vapor film is taken to be thin enough that its curvature can be neglected. Further, the vapor film is assumed to have a linear temperature distribution at all times; Rooney [7] accounted for the effect of pressure on the vapor's temperature distribution and found only a slight influence.

The describing one-dimensional equations in spherical coordinates for the incompressible liquid are the continuity equation

$$\partial(r^2 v)/\partial r = 0 \tag{1}$$

and the momentum equation

$$\rho(\partial v/\partial t + 1/2\,\partial v^2/\partial r) = -\partial P/\partial r. \tag{2}$$

At the liquid–vapor interface conductive heat flow vaporizes liquid so that

$$\dot{M} = k(T_w - T_s)/\delta\Lambda$$

where

$$M = \delta\rho_v P/P_\infty \tag{3}$$

which gives

$$\dot{M} = CP/M. \tag{4}$$

Initially,

$$\delta(t=0) = \delta_0 \tag{5a}$$

$$P(t=0) = P_\infty \tag{5b}$$

$$v(r, t=0) = 0. \tag{5c}$$

Equation (1) shows that

$$r^2 v = f(t). \tag{6}$$

Substitution of this result into equation (2) and integration from $r = R+\delta$ to $r = \infty$ gives

$$(P-P_\infty)/\rho = \dot{f}/(R+\delta) - 1/2 f^2/(R+\delta)^4. \tag{7}$$

At the interface continuity requires that

$$\dot{M} = \rho(\dot{\delta} - v). \tag{8}$$

Substitution of equations (3) and (6) into equation (8), upon rearrangement and use of equation (4), gives

$$f/(R+\delta)^2 = P_\infty C(1-\rho_v P/\rho P_\infty)/M\rho_v - (MP_\infty/\rho_v P^2)\dot{P}. \tag{9}$$

Taking the time derivative of equation (9) gives

$$\begin{aligned} \dot{f}/(R+\delta)^2 - 2f\dot{\delta}/(R+\delta)^3 = &-(P_\infty C/\rho_v)(1-\rho_v P/\rho P_\infty)M^{-2}\dot{M} \\ &-(C/\rho M)\dot{P} \\ &-(P_\infty/\rho_v)(P^{-2}\dot{P}\dot{M} + MP^{-2}\ddot{P} - 2P^{-3}\dot{P}\dot{P}M). \end{aligned} \tag{10}$$

Upon substituting equations (3), (4), (7), and (9) into equation (10) and rearranging into a dimensionless form it is found that

$$\begin{aligned} &\mathrm{d}^2p/\mathrm{d}\tau^2 + \alpha(1+p\rho_v/\rho)pm^{-2}\,\mathrm{d}p/\mathrm{d}\tau - 2p^{-1}(\mathrm{d}p/\mathrm{d}\tau)^2 \\ &+ p^2(p-1)m^{-1} + \alpha^2(1-p\rho_v/\rho)m^{-4}p^3 \\ &- B[\alpha^2(1-p\rho_v/\rho)(1{\cdot}5 + 0{\cdot}5p\rho_v/\rho)m^{-3}p^2 \\ &\quad - \alpha(3-p\rho_v/\rho)m^{-1}\,\mathrm{d}p/\mathrm{d}\tau \\ &\quad + 1{\cdot}5mp^{-2}(\mathrm{d}p/\mathrm{d}\tau)^2] = 0. \end{aligned} \tag{11}$$

Inasmuch as attention is focused on initially thin films, the parameter B is quite small and the terms for which it is a coefficient can be neglected. Also, $\rho_v/\rho \ll 1$ so that equations (11) and (4) can be finally expressed in dimensionless form as

$$\begin{aligned} &\mathrm{d}^2p/\mathrm{d}\tau^2 + \alpha m^{-2}p\,\mathrm{d}p/\mathrm{d}\tau - 2p^{-1}(\mathrm{d}p/\mathrm{d}\tau)^2 \\ &+ p^2(p-1)m^{-1} + \alpha^2 m^{-4}p^3 = 0 \end{aligned} \tag{12}$$

and

$$\mathrm{d}m/\mathrm{d}\tau = \alpha p/m. \tag{13}$$

Initial conditions are

$$p(\tau = 0) = 1 = m(\tau = 0). \quad (14)$$

Equation (8) with the stipulation that the liquid is initially stagnant gives

$$\dot{M} = \rho\dot{\delta}.$$

Use of equation (3) in this relation shows that

$$\dot{M} = (\rho P_\infty/\rho_v)(P^{-1}\dot{M} - MP^{-2}\dot{P}).$$

But equation (4) allows this to be simplified to

$$\dot{P} = CP^2(1 - \rho_v P/\rho P_\infty)/M^2$$

which in dimensionless terms can be written as

$$\mathrm{d}p(\tau = 0)/\mathrm{d}t = \alpha(1 - \rho_v/\rho).$$

Again because $\rho_v/\rho \ll 1$,

$$\mathrm{d}p(\tau = 0)/\mathrm{d}\tau = \alpha. \quad (15)$$

The dimensionless form of equation (3) gives

$$\delta/\delta_0 = m/p. \quad (16)$$

Equations (12)–(15) must be solved to determine the dimensionless film pressure (p), thickness (δ/δ_0), and mass per unit area (m). Unfortunately, equation (12) is markedly nonlinear so that recourse to numerical methods is unavoidable for the general solutions sought.

However, a transformation enables the magnitudes of the pressure excursions to be analytically determined for some limiting cases. Let $z = 2m^{3/2}/3\alpha$ and $y = \log_e(p)$. Then equation (12) becomes

$$z^2\,\mathrm{d}^2y/\mathrm{d}z^2 + (z/3)\,\mathrm{d}y/\mathrm{d}z + z^2(1 - \mathrm{e}^{-y}) = -4/9 \quad (17)$$

while equations (14) and (15) become

$$y(z_0) = 0 \quad (18)$$

$$\mathrm{d}y(z_0)/\mathrm{d}z = \alpha \quad (19)$$

where $z_0 = 2/3\alpha$. Equation (17) has only one nonlinear term which can easily be linearized to obtain approximate solutions. This advantage was gained at the sacrifice of eliminating time from equation (12), however. While expression of dimensionless pressure in terms of dimensionless vapor mass per unit area can be accurately accomplished and the maximum pressure determined, temporal behavior is still best obtained from equation (12) by numerical means.

SOLUTIONS

The importance of the parameter α is displayed in equation (15) where it is seen to act as a forcing function, causing the film pressure's dimensionless time derivative to depart from zero and causing pressure to undergo a subsequent excursion. With equation (13) as a guide, it is realized that α can be interpreted as the initial dimensionless rate of vaporization. This understanding suggests that small pressure excursions would be associated with small vaporization rates or small α, and that large pressure excursion would be associated with large vaporization rates or large α.

Small excursions

The small excursions that would be expected when α is small give small values of y so that $1 - \mathrm{e}^{-y} \approx y$, allowing equations (17)–(19) to be linearized to

$$z^2\,\mathrm{d}^2y/\mathrm{d}z^2 + (z/3)\,\mathrm{d}y/\mathrm{d}z + z^2 y = -4/9$$
$$y(z_0) = 0$$
$$\mathrm{d}y(z_0)/\mathrm{d}z = \alpha$$

whose solution is given by Murphy [8] as

$$y = [c_1 J_{-1/3}(z) + c_2 J_{1/3}(z)]z^{1/3} + az^{1/3}J_{-1/3}(z)\log_e(z) + F(z) \quad (20)$$

where

$$a = 2^{2/3}\Gamma(2/3)/3$$

$$F = z^2 \sum_{n=0}^{\infty} b_{2n} z^{2n}$$

$$b_{2n} = (n + 5/6)\Gamma(2/3)/[(-4)^n(n+1) \times (12n + 7n + 4)n!\Gamma(n + 5/3)].$$

This linearized solution shows the general nature of the results, but its complexity is great enough that it is advantageous to examine the limiting case of $\alpha \to 0$. For such a case z is very large with, according to Abramowitz [9],

$$\lim_{z\to\infty} J_n(z) = (2/\pi z)^{1/2}\cos(z - \pi/4 - n\pi/2)$$

and

$$\lim_{z\to\infty} z^m \log_e(z) = 0 \quad \text{for} \quad m < 0$$

with

$$\lim_{z\to\infty} F(z) = 0$$

as well. The solution for very small α can, therefore, be approximated as

$$y = (c_3 \sin z + c_4 \cos z)z^{-1/6}.$$

Consideration of the initial conditions then gives

$$y = \alpha(3\alpha z/2)^{-1/6}\sin(z - 2/3\alpha) \quad (21)$$

which has

$$y_{\max} \approx \alpha$$

at

$$z \approx 2/3\alpha + \pi/2$$

from which it follows that

$$p_{\max} \approx 1 + \alpha$$

at

$$m \approx (1 + 3\pi\alpha/4)^{2/3}. \quad (22)$$

The pressure excursions in this limiting case are small enough that the vapor mass per unit area increase specified by equation (13) is essentially unaffected, giving

$$\mathrm{d}m/\mathrm{d}\tau \approx \alpha/m$$

so that

$$m^2 \approx 1 + 2\alpha\tau. \quad (23)$$

Equations (22) and (23) show that the maximum pressure excursion occurs at

$$\tau \approx \pi/2. \quad (24)$$

Further, equation (23) substituted into equation (21) shows pressure's temporal behavior at large times to be the lightly damped oscillation

$$p = 1+\alpha(1+2\alpha\tau)^{-1/8}\sin(\omega\tau)$$

where $\omega = 2[(1+2\alpha\tau)^{3/4}-1]/3\alpha\tau$ is a dimensionless frequency. Accordingly, at very small times

$$\omega = 1 \tag{25}$$

while at large times the oscillation's dimensionless frequency and amplitude gradually decrease as $\omega = (128/81\alpha\tau)^{1/4}$ and $\alpha/(2\alpha\tau)^{1/8}$, respectively.

Equation (16), together with the realization that m is little affected by pressure excursions when α is small, shows that the ratio of film thicknesses with and without pressurization has extreme values given by

$$\delta_{\text{pressure}}/\delta_{\text{no pressure}} \approx 1 \pm \alpha. \tag{26}$$

Thus, if there is to be less than a 10 per cent influence of pressure excursions upon film thickness, it is necessary that $\alpha < 0{\cdot}1$.

Large excursions

The large pressure excursions that would be expected when α is large differ in some important ways from the small pressure excursions. The general qualitative behavior deduced previously from a linearized solution is correct for large α only at large times after the excursions have been greatly damped. At small times pressure and y would be very large so that $1-e^{-y} \approx 1$ in equation (17). Thus the limiting case of $z \to 0$ in the linearized solution, equation (20), gives an incorrect result for $F(z)$.

Focusing on the very first excursion for $\alpha \to \infty$ and with the idea that y is large gives equations (17)–(19) as

$$z^2\,d^2y/dz^2 + (z/3)\,dy/dz = -4/9 - z^2$$

with

$$y(z_0) = 0$$
$$dy(z_0)/dz = \alpha$$

whose solution is

$$y = c_5 + c_6 z^{2/3} + (2/3)\log_e(z) - 3z^2/8.$$

Consideration of the initial conditions gives

$$y = 1/3\alpha^2 + (9/32\alpha^4)^{1/3}z^{2/3} + (2/3)\log_e(z) - 3z^2/8$$

which, again, is applicable only to the very first excursion. In the limiting case of $\alpha \to \infty$, this gives

$$y_{\max} = -1/3 + \log_e(2\alpha^2)^{1/3}$$

at

$$z = (8/9)^{1/3}$$

from which it follows that

$$p_{\max} = (2/e)^{1/3}\alpha^{2/3} = 0{\cdot}906\alpha^{2/3}$$

at

$$m = 2^{1/3}\alpha^{2/3}.$$

Inasmuch as a very small time has elapsed, the liquid has not displaced appreciably and this first pressure excursion occurs at nearly constant film thickness. Equations (13) and (16) taken together then give

$$dm/d\tau = \alpha.$$

Thus,

$$m = 1+\alpha\tau$$

and the time at which the pressure reaches its first maximum is

$$\tau = (2/\alpha)^{1/3}. \tag{27}$$

Numerical method

The general case was solved by a numerical method applied to equation (12). All calculations were executed on an H635 digital computer, using the MIMIC program [10]. The numerical solutions were run until the film pressure had undergone several cycles of oscillation and had exhibited behavior which was predictable by the analytical solution for small excursions.

DISCUSSION

The results of the calculations for maximum film pressure are shown in Fig. 2. There it is seen that the limiting solutions for large and small values of the parameter α are in good agreement with the complete numerical solution. It must be remembered, however,

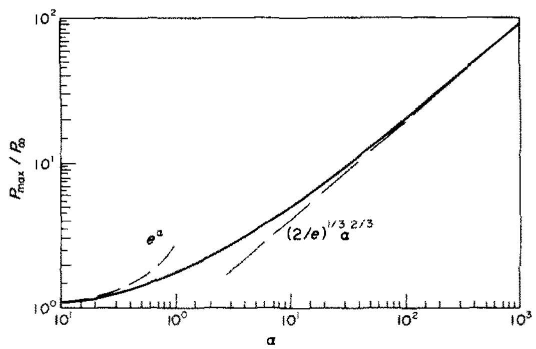

FIG. 2. Dimensionless maximum pressure vs the dimensionless parameter α.

that these solutions neglect liquid compressibility and the dependence of saturation temperature upon pressure. As a consequence the maximum pressures found in this study must be regarded as upper bounds, particularly when α is large. It must also be remembered that there is a lower limit on the initial film thickness which depends upon such factors as geometry, fluid properties, and the specific physical situation encountered—the film does not begin with a zero initial thickness.

To illustrate the application of these results, consider a 500C sphere of 0·3 cm radius suddenly immersed in a 100C pool of saturated water with an initial vapor film thickness of 3×10^{-3} cm. Under these conditions $\alpha = 0{\cdot}6$ and $A = 3{\cdot}5 \times 10^4\,s^{-1}$. From Fig. 2 the predicted maximum pressure in the vapor film is 1·5 atm, representing a maximum pressure excursion of 1/2 atm. This maximum pressure would occur after 45×10^{-6} s, according to equation (24), and pressure would oscillate with a period of 180×10^{-6} s, according to equation (25). It can be shown in a manner paralleling the earlier developments of this study that pressure varies with distance from the sphere as

$$[P(r)-P_\infty]/[P(R)-P_\infty] = R/r.$$

Therefore, a pressure transducer located 2 cm from the sphere would indicate a maximum pressure excursion of only 1/15 atm which is substantially smaller than in the vapor film. While the present study pertains to a saturated liquid and a spherical geometry, the numerical results are still in qualitative agreement with the experimental observation of Board *et al.* [11] for a subcooled liquid and a more nearly plane geometry.

Film pressure as a function of time is depicted in Fig. 3. There the predicted $t^{-1/8}$ decay and slowly decreasing frequency of oscillation is apparent at large times.

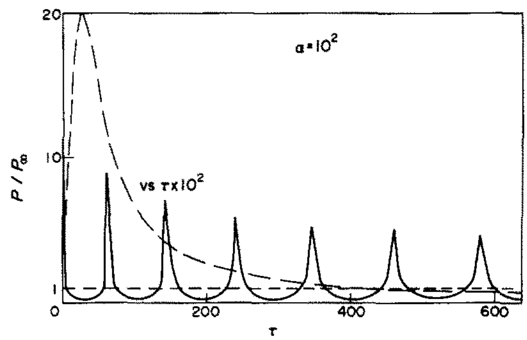

FIG. 3. Dimensionless pressure vs dimensionless time.

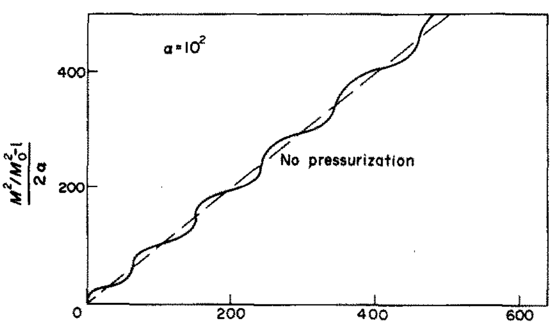

FIG. 4. Dimensionless vapor mass per unit area vs dimensionless time.

Variation of vaporized mass per unit area is displayed in Fig. 4. While there are interesting deviations from classical results, it still can be said with fair accuracy that m^2 varies linearly with time. This is in contrast to the behavior of the vapor film thickness in Fig. 5. Inasmuch as many experimental studies rely upon a measurement of film thickness to determine growth constants, it is clear that substantial error could be incurred if a linear increase in the square of the film

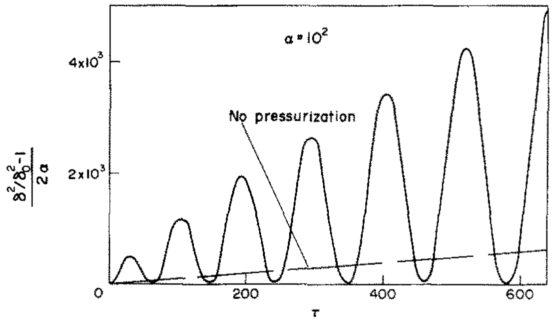

FIG. 5. Dimensionless film thickness vs dimensionless time.

thickness is assumed without consideration of possible pressure excursions. Thus, the assumption of an incompressible vapor and a constant pressure is seen to be substantially in error in some important situations.

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